PROBLEM 1: WEIGHT, PULLEY, AND ROD ASSEMBLY

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A vertical load $P$ is applied at the end of rigid rod B-C.

![Figure 1. Weight, pulley, and rod](image)

REQUIRED

(a) Neglecting the weight of the rod, express the angle $\theta$ corresponding to the equilibrium position in terms of $P$, $l$, and the counterweight $W$.

(b) Determine the value of $\theta$ corresponding to equilibrium when $P = 500$ N, $W = 300$ N, and $l = 500$ mm

(c) Is the answer for part (b) statically stable, statically unstable, or statically neutral?

SOLUTION

Part (a), equilibrium position in terms of $P$, $W$, and $l$

Define position parameter $= \theta$, and its datum (zero potential energy) at $\theta = 0$. The height of the weight $W$ is denoted by $U_W$. Define $U_W = 0$ at $\theta = 0$. The displacement downward of point B is denoted by $U_P$. Define datum $U_P = 0$ at $\theta = 0$.

By geometry, $U_W = \text{distance } |AB| = 2l\sin(\theta/2)$

By geometry, $U_P = l\sin(\theta)$

$V = \text{total potential energy}$

$= WU_W - PU_P$

$= 2W\sin(\theta/2) - P\sin(\theta)$

$dV/d\theta = W\cos(\theta/2) - P\cos(\theta)$

At equilibrium, $dV/d\theta = 0$
\[ W\cos(\frac{\theta}{2}) - Pl\cos(\theta) = 0 \]

\[ (W/P)\cos(\frac{\theta}{2}) - \cos(\theta) = 0 \]

\[ \theta = \cos^{-1}\left((W/P)\cos(\theta/2)\right) \]  (1)

By iteration of equation 1, \( \theta \) corresponding to equilibrium is shown in Figure 2.

Figure 2. Equilibrium position

Part (b): Equilibrium for \( P = 500\text{N}, W = 300\text{N}, l = 500 \text{mm} \)

\[ W/P = 0.6. \text{ By iteration of equation (1), } \theta = 1.02 \text{ rad} (= 58^\circ) \]

Part (c) Stability

\[ \frac{d^2V}{d\theta^2} = -0.5Wl\sin(\theta/2) + Pl\sin(\theta) \]

\[ \frac{d^2V}{d\theta^2} \bigg|_{\theta = 1.02} = -0.5(300\text{N})(0.500\text{m})\sin(1.02/2) + (500\text{N})(0.500\text{m})\sin(1.02) \]

\[ = 176\text{Nm} > 0 \Rightarrow \text{statically stable.} \]
**Problem 2: Rods and Gears**

**Given**
Two uniform rods, $AB$ and $CD$, are attached to gears of equal radii, as shown. The picture is an elevation, so gravity acts downward in the plane of the page. The masses of rods $AB$ and $CD$ are $m_{AB} = 350\text{g}$ and $m_{CD} = 600\text{g}$, respectively.

![Figure 3. Rods and gears configuration](image)

**Required**
Determine the positions of equilibrium of the system. State in each case whether the equilibrium is stable, unstable, or neutral.

**Solution**
Position parameter is $\theta$. $V_g = 0$ at $\theta = 0$.

$$V(\theta) = -m_{AB}(l/2)\sin(\theta)g + m_{CD}(l/2)(1 - \cos(\theta))g$$

At equilibrium,

$$\frac{dV}{d\theta} = -m_{AB}(l/2)\cos(\theta)g + m_{CD}(l/2)\sin(\theta)g = 0$$

$$-m_{AB}\cos(\theta) + m_{CD}\sin(\theta) = 0$$

$$\frac{m_{AB}}{m_{CD}} = \tan(\theta)$$

$$\theta = \tan^{-1}\left(\frac{m_{AB}}{m_{CD}}\right) - n\pi, \quad n = 0, 1$$

$$= \tan^{-1}\left(\frac{350}{600}\right) - n\pi$$

$$= 0.528, -2.61 \text{ (or equivalently, } 30.3^\circ, -150^\circ)$$

$$\frac{d^2V}{d\theta^2} = m_{AB}(l/2)\sin(\theta)g + m_{CD}(l/2)\cos(\theta)g$$

$$\frac{d^2V}{d\theta^2}|_{\theta_{eqm}} = 3.41l \text{ or } -3.41l \text{ for } \theta = 0.52 \text{ and } -2.61, \text{ respectively}$$

Stable equilibrium at $\theta = 0.528 \text{ rad (30.3°)}$, unstable equilibrium at $\theta = -2.61 \text{ rad (-150°)}$