Problem 2: A 2 kg mass rests on a flat horizontal bar. The bar begins rotating in the vertical plane about O with a constant angular acceleration of 1 rad/s². The mass is observed to slip relative to the bar when the bar is 30° above the horizontal. What is the static coefficient of friction between the mass and the bar? Does the mass slip toward or away from O?

Solution From Newton's second law for the radial component

\(-mg \sin \theta \pm \mu_s N = -mR\alpha^2\), and for the normal component:

\(N - mg \cos \theta = mR\alpha\). Solve, and note that

\(\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} = 1 = const\), \(\omega^2 = 2\theta\), since \(\omega(0) = 0\), to obtain

\(-g \sin \theta \pm \mu_s(g \cos \theta + R\alpha) = -2R\theta\). For \(\alpha = 1\), \(R = 1\), this reduces to

\(\pm \mu_s(1 + g \cos \theta) = -2\theta + g \sin \theta\). Define the quantity \(F_R = 2\theta - g \sin \theta\). If \(F_R > 0\), the block will tend to slide away from O, the friction force will oppose the motion, and the negative sign is to be chosen. If \(F_R < 0\), the block will tend to slide toward O, the friction force will oppose the motion, and the positive sign is to be chosen. The equilibrium condition is derived from the equations of motion:

\(\text{sgn}(F_R)\mu_s(1 + g \cos \theta) = (2\theta - g \sin \theta)\). From which

\(\mu_s = \frac{\text{sgn}(F_R)2\theta - g \sin \theta}{1 + g \cos \theta} = 0.406\). Since \(F_r = -3.86 < 0\), the block will slide toward O.